Computational Intelligence

Unit # 12

What is Fuzzy Logic

- Definition of Fuzzy
  - Fuzzy: "not clear, distinct, or precise; blurred"
- Definition of Fuzzy Logic
  - A form of knowledge representation suitable for notations that cannot be defined precisely but which depend upon their contexts.
- The term was coined by Lotfi Zadeh in 1965 with his mathematics of fuzzy set theory.

Successful Applications

- Automatic Control of dam gates for hydroelectric-power plants
- Camera aiming
- Compensation against vibration in camcorders
- Cruise-control for automobiles
- Controlling air-conditioning systems
- …..
- Many others

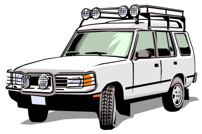
Examples of Linguistic Impression

- How was the weather like yesterday?
  - Oh! It was rainy with 98% humidity and hot with temperature of 35.5 deg C
  - Oh! It was <u>very</u> humid and <u>really</u> hot.

* Source: University Malaysian Pahang

Examples of Linguistic Impression (Cont'd)

- When you are at 10 meters from the junction start braking at 50% pedal level.



- When you are near the junction, start braking slowly.

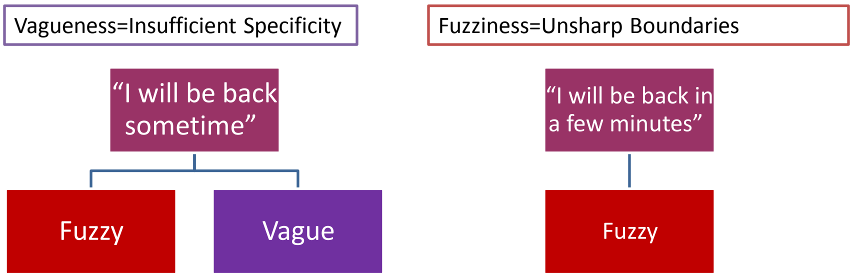
* Source: University Malaysian Pahang

Uncertainty vs Vagueness*

- Certainty – degree of belief
  - There is a 50% probability of rain today
  - I am 30% sure the patient is suffering from pneumonia
- Fuzziness – the degree to which an item belongs to a category
  - The man is tall
  - Move the wheel slightly to the left
  - The patient's lungs are highly congested

* Source: Susan Bridges @ Mississippi State University

Fuzziness Vs. Vagueness*

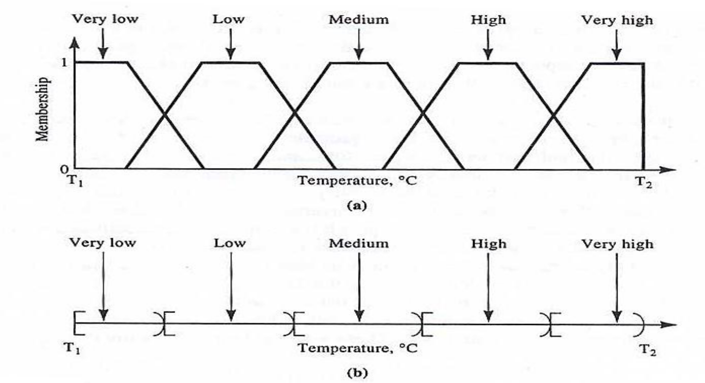


* Source: Raphael Steinberg @ Technion University

Bivalent Logic

- In classical logic, which is often described as Aristotelian logic, there are two possible truth values: propositions are either true or false.
- Such systems are known as **bivalent logics because they involve two logical values.**
- The logic employed in Bayesian reasoning and other probabilistic models is also bivalent: each fact is either true or false, but it is often unclear whether a given fact is true or false.
- Probability is used to express the likelihood that a particular proposition will turn out to be true.

Crisp vs. Fuzzy Variable

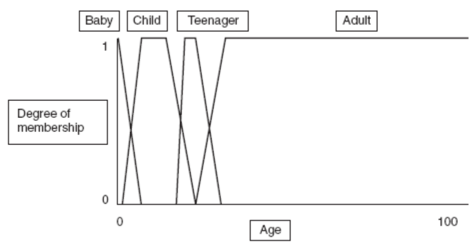


Temperature in the range $[T_1, T_2]$ conceived as: (a) a fuzzy variable; (b) a traditional (crisp) variable.

Example of a Fuzzy Variable

$$M_B(x) = \begin{cases} 1 - \frac{x}{2} & for\ x \le 2 \\ 0 & for\ x > 2 \end{cases}$$

$$M_C(x) = \begin{cases} \frac{x-1}{6} & for\ x \le 7 \\ 1 & for\ x > 7\ and\ x \le 8 \\ \frac{14-x}{6} & for\ x > 8 \end{cases}$$



- We represent a fuzzy set using a list of pairs, where each pair represents a value and the fuzzy membership value for that value.
- For example, we might define *B*, the fuzzy set of babies as follows:
  - *B* = {(0, 1), (2, 0)}
- Similarly,we could define the fuzzy set of children,*C*, as follows:
  - *C* = {(1, 0), (7, 1), (8, 1), (14, 0)}

Fuzzy Sets

- In traditional two-valued set theory, an element either belongs to a set or not. That is, set membership is precise.
- In fuzzy sets, an element belongs to a set to a degree, indicating the certainty (or uncertainty) of membership.

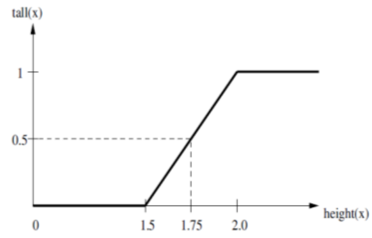
Membership Function

- The function is used to associate a degree of membership of each of the elements of the domain to the corresponding fuzzy set.
- Conditions
  - A membership function must be bounded from below 0 and from above by 1.
  - The range of a membership function must therefore be [0, 1].
  - For each $x \varepsilon X$, $\mu_A(x)$ must be unique. That is, the same element cannot map to different degrees of membership for the same fuzzy set.

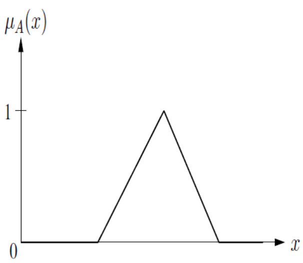
Illustration of *tall* Membership Function

- A possible membership function for *tall* fuzzy set can be defined as

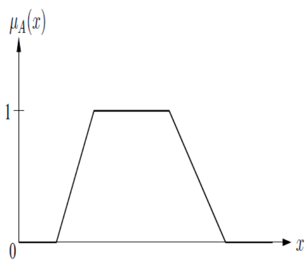
$$\text{tall}(x) = \begin{cases} 0 & \text{if length}(x) < 1.5 \\ (\text{length}(x) - 1.5) * 2 & \text{if } 1.5 < \text{length}(x) < 2 \\ 1 & \text{if length}(x) > 2 \end{cases}$$



Other Function Types



(a) Triangular Function



(b) Trapezoidal Function

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \le \alpha_{min} \\ \frac{x-\alpha_{min}}{\beta-\alpha_{min}} & \text{if } x \in (\alpha_{min}, \beta] \\ \frac{\alpha_{max}-x}{\alpha_{max}-\beta} & \text{if } x \in (\beta, \alpha_{max}) \\ 0 & \text{if } x \ge \alpha_{max} \end{cases}$$

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \le \alpha_{min} \\ \frac{x-\alpha_{min}}{\beta_1-\alpha_{min}} & \text{if } x \in [\alpha_{min}, \beta_1) \\ \frac{\alpha_{max}-x}{\alpha_{max}-\beta_2} & \text{if } x \in (\beta_2, \alpha_{max}) \\ 0 & \text{if } x \ge \alpha_{max} \end{cases}$$

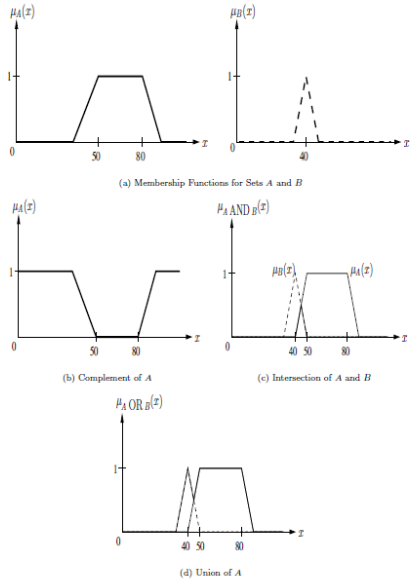
Fuzzy Operators

- **Equality:** Two fuzzy sets A and B are equal if and only if the sets have the same domain, and $\mu_A(x) = \mu_B(x)$ for all $x \in X$.
- **Complement of fuzzy set:** Let A^c denote the complement of set A. Then for all $x \in X$, $\mu_{A^c}(x) = 1 - \mu_A(x)$.

Fuzzy Operators (Cont'd)

- Intersection of fuzzy sets: If A and B are two fuzzy sets, then
  – Min-operator: $\mu_{A \cap B}(x)=\min\{\mu_A(x),\mu_B(x)\}, \forall x \in X$
  – Product Operator: $\mu_{A \cap B}(x)=\mu_a(x)\mu_B(x), \forall x \in X$
- Union of fuzzy sets: If A and B are two fuzzy sets, then
  – Max-operator: $\mu_{A \cup B}(x)=\max\{\mu_B(x),\mu_B(x)\}, \forall x \in X$
  – Summation Operator: $\mu_{A \cup B}(x)=\mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \forall x \in X$

Illustration of Fuzzy Operators



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Exercise 1

Consider the two fuzzy sets:

$$\text{long pencils} = \{pencil1/0.1, pencil2/0.2, pencil3/0.4, pencil4/0.6, pencil5/0.8, pencil6/1.0\}$$

$$\text{medium pencils} = \{pencil1/1.0, pencil2/0.6, pencil3/0.4, pencil4/0.3, pencil5/0.1\}$$

(a) Determine the union of the two sets.

(b) Determine the intersection of the two sets.

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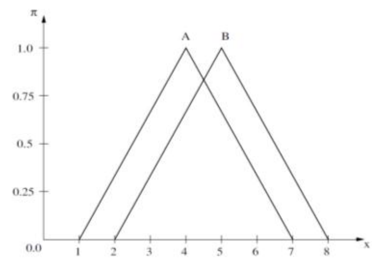
Exercise 2

- Consider the membership function of two fuzzy sets, A and B, as given in the figure.
  - Draw the membership function for the fuzzy set C = A ∩ B^c, using the min-operator.
  - Compute $\mu_C(5)$.

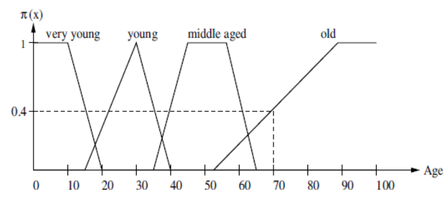


Fuzzy Reasoning

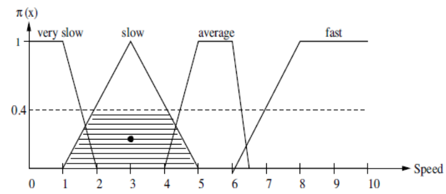
- A fuzzy reasoning system consists of three other components, each performing a specific task in the reasoning process:
  - Fuzzification
  - Inferencing
  - Defuzzification

Fuzzy Reasoning: Example 1

- Rule
  - If *Age* is *Old* the *Speed* is *Slow*
- What can be said about *Speed* if *Age* has the value of 70?



(a) *Age* Membership Functions



(b) *Speed* Membership Functions

Fuzzy Reasoning: Example 2

Let us suppose that we are designing a simple braking system for a car, which is designed to cope when the roads are icy and the wheels lock.

The rules for our system might be as follows:

Rule 1 IF pressure on brake pedal is medium
THEN apply the brake

Rule 2 IF pressure on brake pedal is high
AND car speed is fast
AND wheel speed is fast
THEN apply the brake

Rule 3 IF pressure on brake pedal is high
AND car speed is fast
AND wheel speed is slow
THEN release the brake

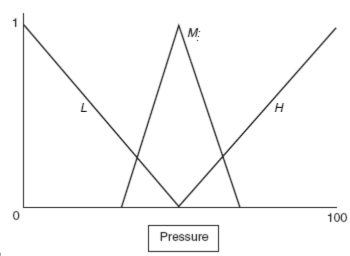
Rule 4 IF pressure on brake pedal is low
THEN release the brake

For this simple example, we will assume that brake pressure is measured from 0 (no pressure) to 100 (brake fully applied). We will define brake pressure as having three linguistic values: high (H), medium (M), and low (L), which we will define as follows:

$H = \{(50, 0), (100, 1)\}$

$M = \{(30, 0), (50, 1), (70, 0)\}$

$L = \{(0, 1), (50, 0)\}$



Example 2 (Cont'd)

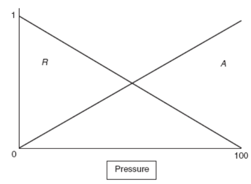
Similarly, we must consider the wheel speed. We will define the wheel speed as also having three linguistic values: slow, medium, and fast. We will define the membership functions for these values for a universe of discourse of values from 0 to 100:

$S = \{(0, 1), (60, 0)\}$

$M = \{(20, 0), (50, 1), (80, 0)\}$

$F = \{(40, 0), (100, 1)\}$

For the sake of simplicity, we will define the linguistic variable *car speed* using the same linguistic values (*S*, *M*, and *F* for slow, medium, and fast), using the same membership functions. Clearly, in a real system, the two would be entirely independent of each other.



Example 2: Fuzzification

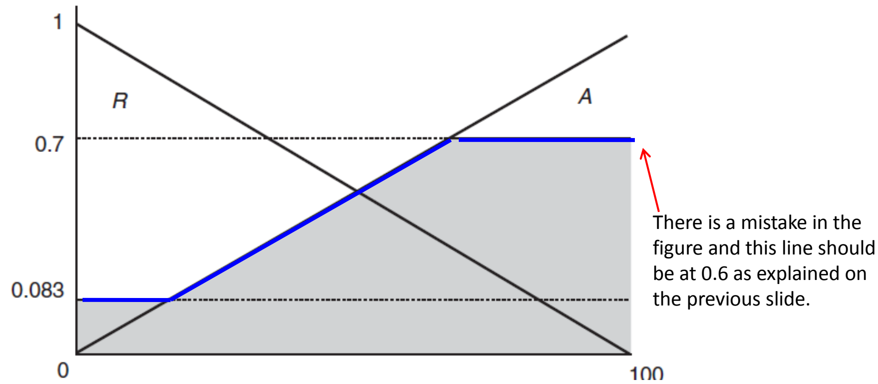
- In a given situation, pressure value is 60, wheel speed is 55, and the car speed is 80.
  - $M_L(60) = 0.0$
  - $M_M(60) = 0.5$
  - $M_H(60) = 0.2$
  - $M_S(55) = 0.083$
  - $M_M(55) = 0.833$
  - $M_F(55) = 0.250$
  - $M_S(80) = 0.0$
  - $M_M(80) = 0.0$
  - $M_F(80) = 0.667$

Example 2: Inferencing

- Fuzzy values obtained from the four rules are:
  - Rule 1: 0.5
  - Rule 2: Min(0.2, 0.667, 0.25) = 0.2
  - Rule 3: Min(0.2, 0.667, 0.083) = 0.083
  - Rule 4: 0

- Apply break should be 0.5 + 0.2 – 0.5*0.2 = 0.6
- Release break should be 0.083.

Example 2: Defuzzification



- Center of Gravity = (5 x 0.083 + 10x0.1 + 15x0.15 +……+ 70x0.6 + 75x0.6 + 80x0.6 + …. + 100x0.6)/ (0.083 + 0.1 + 0.15 + …… + 0.6)
- Center of Gravity = 63.97